transfer through the separation wall is taking place from inner to outer flow. For all other values of ω, the heat transfer direction at the separation surface is from outer to inner flow. The curve for $\eta = -1.0$ displays a similar behavior, with the difference being that the ω interval shifts to a new position of $0.4622 \leq \omega \leq 0.5604$.

Figure 3

These curves show the variation of the ratio of the Nusselt numbers for constant heat flux $(Nu_o)_H$ to the Nusselt number for constant wall temperature $(Nu_o)_T$, for four values of heat exchanger number η (i.e., 1, 10, -1, -10), corresponding to parallel and counterflow arrangements. It can be observed that for parallel flow arrangements this ratio is always greater than one. In the limiting cases of $\omega = 0$ and $\omega = 1.0$, corresponding to the case of a simple pipe, the ratio obtained has a value of 1.1703. The classical value of this ratio is 1.1930. This discrepancy is attributed to the fact that the analysis presented herein includes only the first approximation from the "iteration method" used. Moreover, there is a particular value of ω for parallel flow arrangement (i.e., $\omega = 0.4340$) for which a phenomenon occurs. When the core size is smaller than this number, the ratio of the Nusselt numbers for $\eta = 1.0$ is smaller than the ratio of the Nusselt numbers for $\eta = 10.0$. When the core size is greater than 0.4340, the opposite phenomenon occurs. In case of a counterflow arrangement, the curve with $\eta = 1.0$ has one vertical asymptote at $\omega = 0.5536$, and the curve with $\eta = -10.0$ has another vertical asymptote at $\omega = 0.3849$. The physical significance of these asymptotes is such that they correspond to ω values of Fig. 1 for which $(Nu)_o$ vanishes.

Figure 4

These curves show the variation of the ratio of the inner Nusselt number for constant heat flux $(Nu_i)_H$ to constant wall temperature $(Nu_i)_T$ for four values of heat exchanger number η (i.e., 1, 10, -1, -10), corresponding to parallel and counterflow arrangements. As in the previous limiting case of $\omega = 1.0$, the ratio has the same value of 1.1703 representing the simple flow situation. However, in the limiting case of $\omega = 0$, the ratio is approaching a value of zero. The curve with $\eta = 1.0$ has an asymptote at $\omega = 0.5483$, and the curve with $\eta = 10.0$ has a asymptote at $\omega = 0.4111$. The meaning of this behavior is that, at some values of ω, these curves are crossing the horizontal axis. That is, at these ω values, the heat transfer at the separating surface is changing its direction. Moreover, for both cases where $\eta = 1.0$ and $\eta = 10.0$, the ratio has an optimum value of 0.85 at $\omega = 0.0350$. In case of counterflow arrangements, the curve with $\eta = -10.0$ has an asymptote at $\omega = 0.2667$ and the curve with $\eta = -1.0$ has an asymptote at $\omega = 0.4622$.

Acknowledgment

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Friction Coefficients for Flow in Pipes with Mass Injection and Extraction

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Theme

FUNCTIONS for friction coefficients are presented for flow in a pipe with mass injection or extraction. The expressions are valid for compressible flows (Mach numbers as high as 1) with very large mass injection or extraction rates (radial Reynolds numbers up to 20,000). In order to develop these relationships, a porous pipe with air injection and extraction was studied experimentally and numerically. For the numerical model, the compressible, axisymmetric, Navier-Stokes equations were solved using MacCormack's explicit finite-difference method. No boundary-layer assumptions were made. Experimentally, pressure variations axially and velocity variations axially and radially inside the porous pipe were measured. The experimental data were used to show that numerical solutions gave valid results. Friction coefficient expressions were developed using the numerical data. They were shown to give excellent results when used in a one-dimensional model to predict compressible flow dynamics resulting from mass injection/extraction.

Contents

Figure 1 is a representation of the system studied experimentally and numerically. During the experiment a high-pressure air source (6.9×10^5 N/m^2 gage) provided air that was injected radially into a porous pipe. The air would then flow axially down the pipe and exit radially through a second section of

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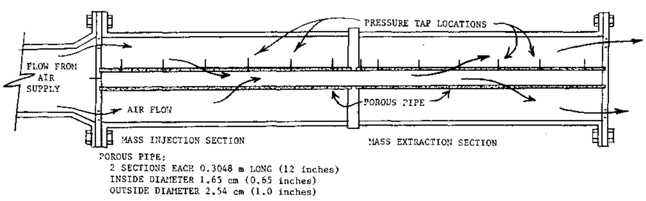

Fig. 1 Porous pipe experimental setup.

porous pipe. Pressure taps installed along the porous pipe were used to measure the axial pressure distribution. Also, seven total pressure tubes were combined to form a rake used for velocity measurements. To measure the effect of mass injection, extraction, and pressure gradient on the flowfield's turbulence, the velocity rake was replaced by a hot-film anemometer.

The flow with mass injection/extraction was numerically modeled to obtain more flow details than could be measured during the experimental portion of the research. The flow was assumed to be compressible and axisymmetric. The Reynolds-averaged continuity, momentum, and energy equations were used to describe the turbulent flowfield. A model proposed by Baldwin and Lomax[2] was used to model the eddy viscosity in the turbulent regions. The Dhawan and Narasimha transition region model[3] was used to model the change from fully laminar to fully turbulent flow. The location of transition was determined from the experimental data. The general solution procedure was that developed by MacCormack[4] known as the Explicit MacCormack Method.

Figure 2 contains the numerically determined and the experimentally measured axial pressures variations for three sample cases. The maximum Mach numbers obtained in the flowfields for the three cases were 0.175, 0.635, and 1.70. For all three cases, axial pressure drops where observed as the flow accelerated due to mass injection ($0.0 \le Z/L \le 0.5$). For the two subsonic cases, the pressure rose as the air decelerated due to mass removal ($0.5 \le Z/L \le 1.0$). For the supersonic case, mass removal caused the air to accelerate (pressure drop) until a normal shock was encountered ($Z/L = 0.64$).After the shock, the pressure increased as the flow slowed down.

Once the numerical solutions were shown to accurately model the flow, the numerical results were used to predict friction coefficients f defined as

$$f = \frac{2\tau_w}{\bar{\rho}\bar{U}^2} \tag{1}$$

where τ_w is the shear stress at the pipe wall, $\bar{\rho}$ is the average density, and $\bar{U}$ is the average axial velocity. Eleven numerical simulations were conducted for pipe aspect ratios varying from $L/D = 12$ to $L/D = 96$ (L is the total system length, and D is the pipe i.d.) and with maximum Mach numbers between 0.1 and 1.0. The flowfields generated by the numerical simulations were used to estimate the friction coefficient.

For the region with mass injection, the least-squares representation of the data is

$$f \times Re = 16(1.2337 - 0.2337 e^{0.0363\, Re_W}) e^{6M^2/5} \tag{2}$$

where M is the Mach number based on the local mean axial velocity, Re_w is the radial Reynolds number, and Re is the axial Reynolds number defined as

$$Re_w = \frac{\rho_0 v_0 D}{\mu}, \qquad Re = \frac{\bar{\rho}\bar{U}D}{\mu}$$

where ρ_0 is the density of the fluid being injected or extracted at the wall, μ is fluid viscosity, and v_0 is the radial velocity at the wall. Equation (2) is valid for laminar flow with Mach numbers as high as 1, axial Reynolds numbers as high as 1,000,000, and any negative radial Reynolds number (mass

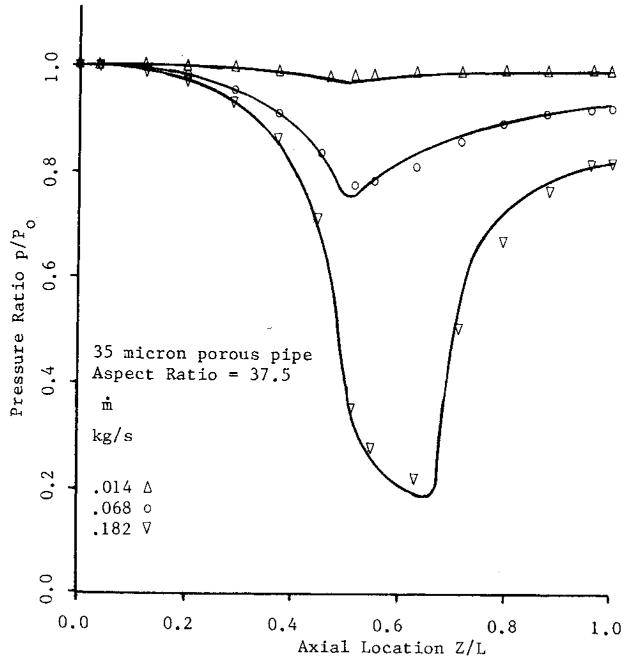

Fig. 2 Numerical and experimental pressure data.

injection). The favorable pressure gradient in the mass injection region influenced the flow to remain laminar, even for axial Reynolds numbers as high as 1,000,000.

The data for the mass extraction region were studied to find a convenient way of representing it functionally. The compressibility effects were assumed to be the same as in the mass injection region. The least-squares fit for the data is

$$f = \frac{0.046}{Re^{1/5}} \left[1 + 55\, Re^{0.1} \left(\frac{v_0}{\bar{U}}\right)^{0.9} \left(\frac{2L_c}{D}\right)^{0.1} e^{6M^2/5} \right] \tag{3}$$

where L_c is the length of the mass extraction region. The ratio v_0/U is referred to as the extraction parameter. This expression predicts the friction coefficient for fully developed turbulent flow in a pipe with mass extraction at the pipe wall. The equation is valid for turbulent flow with Mach numbers as high as 1.0, axial Reynolds numbers up to 1,000,000, and radial Reynolds numbers up to 20,000 (extraction parameters as high as 2.0).

The preceding expressions can be compared with earlier researchers' work. Friction coefficients for the laminar, incompressible, pipe flow with mass injection were first developed by Yuan and Finkelstein in 1956.[5] They predicted that as mass injection increased, the friction coefficient would increase from the noninjection friction coefficient and, with large injection rates, would approach

$$f = \frac{2\pi^2}{Re} \tag{4}$$

This is the same result found by this research for small Mach numbers.

Aggarwal et al.[6] experimentally studied the effect of mass extraction on friction coefficients in porous pipes. For their experiments, the pipe inlet axial Reynolds numbers varied from 11,000 to 101,000. Their experimental results for an axial Reynolds number equal to 80,000 are compared with the results of this research in Fig. 3.

Another comparison can be made. Kinney and Sparrow[7] analytically studied the effect of mass extraction on turbulent flow in a pipe. Their results are also shown in Fig. 3. They limited their work to axial Reynolds numbers varying from 10,000 to 150,000 and extraction parameters up to 0.02. Recall that the data used to develop the model presented in this paper were for axial Reynolds numbers between 20,000 and

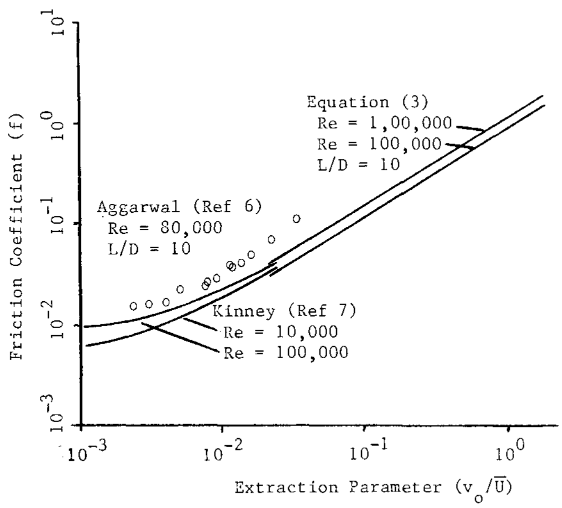

Fig. 3 Comparison with other models.

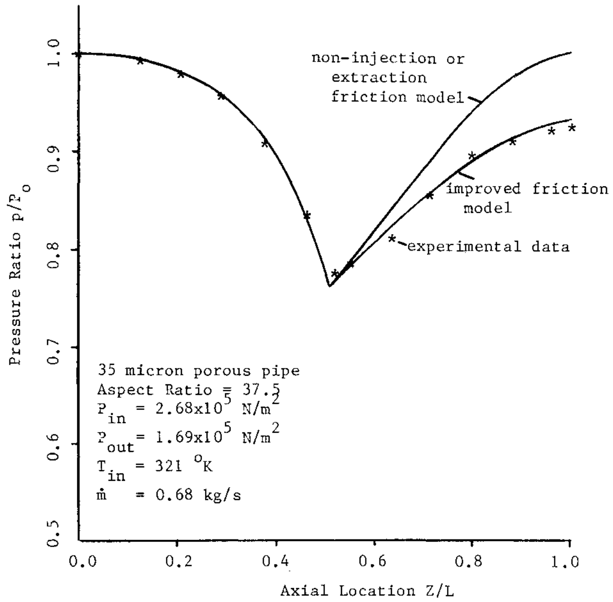

Fig. 4 1-D model with improved friction model.

1,000,000 and for extraction parameters between 0.015 and 2.0. This represents both larger extraction parameters and axial Reynolds numbers than studied by Aggarwal or Kinney.

To test the new friction models, a simplified flow model was studied that assumed the flow was compressible, one-dimensional, adiabatic, and steady. The study conducted compared two friction models. One model assumed that the friction coefficient was the same as used for nonblowing smooth pipe calculations.

$$f(\text{laminar}) = \frac{16}{Re}, \qquad f(\text{turbulent}) = \frac{0.046}{Re^{1/5}} \tag{5}$$

A smooth transition from the laminar region to the turbulent region was used.[3]

The other friction coefficient model used the expressions developed earlier in this paper [Eqs. (2) and (3)]. Figure 4 compares the numerical results from the two friction models with the experimental data. Extremely good results were obtained with the improved friction model.

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Effects of Wetted Wall on Laminar Mixed Convection in a Vertical Channel

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Introduction

THE effects of mass diffusion on natural convection heat transfer have been studied widely for external[1,2] and internal[3] flows. As far as pure forced convection flows are concerned, combined heat and mass transfer in laminar forced convection flow over a flat surface was studied by Chow and Chung.[4] For mixed convection, Santarelli and Foraboschi[5] investigated the influence of natural convection on laminar forced convection in a reacting fluid.

Despite the fact that the influence of combined buoyancy forces of heat and mass diffusion was shown to be relatively important,[1-5] it has not received enough attention. In this Note, consideration is given to the effects of coupled thermal and mass diffusion on laminar forced convection. Particular attention is given to investigating the role of latent heat transfer, in association with the liquid film evaporation from the wetted channel wall, in laminar mixed convection heat transfer.

Analysis

The geometry to be examined is a vertical parallel-plate channel with width $2b$. The channel wall is wetted by thin water films. The loss of evaporated water in the film is compensated by the injection of additional water through the

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